EX. 45: Specific charge of the electron *e/m₀*

1. Objective

Determination of the specific charge of the electron (e/m_0) from the path of an electron beam in crossed electric and magnetic fields of variable strength.

2. Introduction

In this experiment electrons are accelerated in an electric field (neon-filled narrow beam tube with heater 6.3V/0.5A and anode/cathode voltage) and enter a magnetic field at right angles to the direction of motion. The specific charge of the electron is determined from the accelerating voltage, the magnetic field strength and the radius of the electron orbit.

If an electron of mass m_0 and charge e is accelerated by a potential difference U, it attains the kinetic energy:

$$eU = \frac{1}{2} m_0 v^2 \tag{1}$$

where v is the velocity of the electron.

In a magnetic field of strength $\vec{B}$, the Lorentz force acting on an electron with velocity $\vec{v}$ is:

$$\vec{F} = e\vec{v} \times \vec{B} \tag{2}$$

If the magnetic field is uniform, as it is in the Helmholtz arrangement, the electron follows a spiral path along the magnetic lines of force, which becomes a circle of radius r if $\vec{v}$ is perpendicular to $\vec{B}$.

Since the centrifugal force $\frac{m_0 v^2}{r}$ produced is equal to the Lorentz force $\vec{F}$, we obtain

$$v = \frac{e}{m_0} \cdot B \cdot r \tag{3}$$

where B is the absolute magnitude of $\vec{B}$. From equation (1), it follows that

$$\frac{e}{m_0} = \frac{2U}{(Br)^2} \tag{4}$$

Helmholtz coils were used to generate a uniform magnetic field. Helmholtz coils are a pair of identical circular coils that are spaced apart by a distance equal to their radius. The coils are arranged coaxially and are connected such that the current flows in the same direction in both coils. This configuration produces a nearly uniform magnetic field in the region between the coils.

To calculate the magnetic field B, the first and fourth Maxwell equations are used in the case where no time dependent electric fields exist. We obtain the magnetic field strength B_z on the z-axis of a circular current I for a symmetrical arrangements of two coils at a distance a=R from each other:

$$B_z = \frac{\mu_0 \cdot INR^2}{2}\left[\frac{1}{\left(R^2+\left(z-\frac{R}{2}\right)^2\right)^{\frac{3}{2}}} + \frac{1}{\left(R^2+\left(z+\frac{R}{2}\right)^z\right)^{\frac{3}{2}}}\right] \tag{5}$$

with $\mu_0 = 1.257 \cdot 10^{-6} \frac{Vs}{Am}$ and R is radius of coil.

For the Helmholtz arrangement of two coils ($a = R$) with number of turns N, in the centre between the coils ($z = 0$) one obtains

$$B = \left(\frac{4}{5}\right)^{\frac{3}{2}} \mu_0 \frac{IN}{R} \tag{6}$$

Thus from equation (4) one derives

$$\frac{e}{m_0} = 2\left(\frac{5}{4}\right)^3 \frac{UR^2}{\mu_0^2 N^2 I^2 r^2} \tag{7}$$

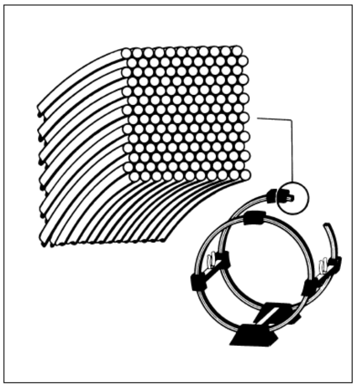
where: U – anode voltage, R – coil radius , N – number of coil turns r – radius of the circle described by the electron beam I - Helmholtz coil supply current

3. Experimental Setup

Fig.1. Experimental setup - By varying the magnetic field (current) and the velocity of the electrons (acceleration and focussing voltage), the radius of the orbit can be adjusted, such that it coincides with the radius defined by the luminous traces.

When the electron beam coincides with the luminous traces, only half of the circle is observable. The radius of the circle is then 2, 3, 4 or 5 cm.

4. Conducting the Experiment



Helmholtz Coils Each of the similar Helmholtz coils is cantilevered off a plastic foot. The coils are positioned by three spacer rails so that their axial spacing is equal to the average coil radius. (The rails can be removed after undoing knurled screws; allowing the coils to be used individually). Two for these rails are provided with a clip holding the narrow-beam tube in the centre of the pair of coils. The sockets of the coil winding are cast into the plastic foot of the coil; and the connecting leads can be used to connect the coils in parallel or series as required. The sockets are numbered (1; 2) to make it easier to wire the coils.

Fig. 2 Winding arrangement of the Helmholtz coils.

Cantilevering the coils from the base allows the number of turns to be counted. The fact that the turns of the individual layers are slightly offset relative to one another (see Fig. 2) must be allowed for when counting. Each coil is wound from copper wire in 14 layers, each of 11 turns, giving the numer of turns $n = 154$.

Additional Parameters:
Helmholtz coil radius: R=0.2 m, $u(R)$=0.01m
Number of Coil Turns: $N = 154$

<u>Narrow Beam Tube</u>

- Glass sphere with two glass tubes covered with plastic caps for fixing in Helmholtz-coils
- Gas filled neon
- Fluorescent distance markers for parallax-free determination of the diameter of the narrow beam
- Plastic end caps for mounting and 4 mm connection sockets with wiring diagram
- Heater 6.3V/0.5A
- Anode/cathode voltage. 150V/-50V
- Piston diameter 170 mm
- Overall length 470mm
- Pressure 0,004 mbar
- Total length 47 cm.

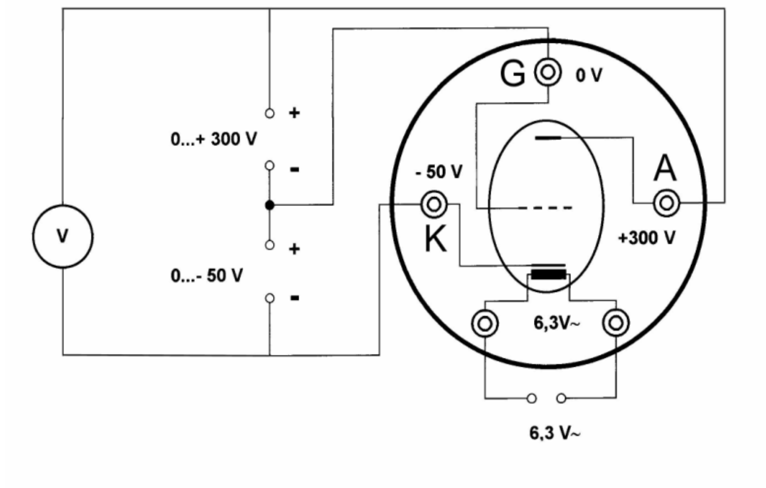


Fig. 3: Electrical circuit of the narrow beam tube.

<u>Electrical connection of the narrow-beam tube</u>

All of the electrical supply voltages for the narrow-beam tube are taken from the mains adapter PHYWE Power supply (regulated DC: 0...12 V, 0,5 A; 0...650 V, 50 mA / AC: 6,3 V, 2 A) . The sockets on the end of the tube are marked so that correct connection to the power supply should not present any problems. Fig. 3 shows the circuit for the setup shown in Fig. 1. The circuit of the narrow-beam tube described above selects a fixed filament voltage of 6.3 V~, whereas the grid voltage supplied to the electron gun and the anode voltage can be adjusted with the aid of the potentiometers fitted in the mains adapter from -50⋯0 V and 0⋯+300 V. The velocity and hence the energy of the electrons of the narrow beam is given by the total accelerating voltage *U* acting between cathode and anode. It is measured with a 300 V⁻ moving-coil instrument.

Starting the Apparatus (performed by the instructor):

1. Verify that the grid voltage and anode voltage on the cathode ray tube power supply are set to zero (turn the potentiometer knobs fully counterclockwise).
2. Turn on the power supply using the switch at the back of the housing. The filament circuit will activate automatically. Wait for at least 1 minute.
3. Turn on the grid voltage and set it to a constant value of 30 V.
4. Turn on the anode voltage and set its initial value to 175 V.
5. Check the grid and anode voltage readings on the connected meters.
6. After approximately 3 minutes, the trace of the electron beam emitted by the cathode ray tube's electron gun should become visible.
7. **Caution!** Before turning off the cathode ray tube power supply, set the grid and anode voltage knobs to zero (fully counterclockwise).
8. On the Helmholtz coil power supply (on the left), check that both current control knobs are set to zero (turned fully counterclockwise). Turn on the Helmholtz coil power supply using the button on the front of the housing, and set the voltage on the power supply to its maximum value. **Do not adjust the voltage during the measurements**—it should remain at its maximum value. While observing the electron beam path, gradually increase the current in the coils until a circular path is achieved.
   The correct alignment of the electron beam relative to the magnetic field is achieved by gently rotating (if needed) the blue tip of the cathode ray tube to form a closed circle instead of a spiral path. This indicates that the electron beam emitted by the electron gun is perpendicular to the magnetic field lines generated by the coils. Additional Parameters

Helmholtz coil radius: R=0.2 m, $u(R)$=0.01m
Number of Coil Turns: N = 154

Measurements

1. Set the anode voltage to U=150 V.
2. By adjusting the current flowing through the coils, record the current value at which the beam hits a step of the ladder placed inside the tube for r=2,3,4,5 cm. For better visibility, the ladder steps are coated with a fluorescent layer.
3. Place the measurement results in Table 1.
4. Repeat the same procedure for anode voltages of 175 V, 190 V, 200 V, 210 V
5. Calculate e/m_0 using formula (7)
6. Attempt to repeat the measurements for different grid voltage values: 20-40V.

Table 1. Measurement results of the current *I* flowing in the coils for different radii of the circles traced by the beam and for different anode voltages

	U = 150V				U = 175V				U = 190V				U = 200V				U = 210V			
r[cm]	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
I [A]																				
e/m_0																				

Control Questions

1. What is the specific charge of an electron and in what units is it measured? What is its tabulated value in SI units?
2. Magnetic induction around a conductor carrying current – Biot-Savart law. What do the magnetic field lines look like around a straight conductor with current, and what about a circular coil with current?
3. When do we say that a magnetic field is uniform? What does it mean? Where can a uniform field be observed (approximately)? How can a uniform magnetic field be obtained?
4. Provide the formula for the Lorentz force acting in a magnetic field on a moving charge. How can the direction of the charge's trajectory curvature be determined based on this force?
5. How to determine the velocity of charge q accelerated by a constant voltage U?
6. How does magnetic induction change along the axis of a circular coil?