Ex. 47 Millikan's Experiment

Objective of the Exercise

Determining the electric unit charge after Millikan and verifying the charge quantification

Introduction

Millikan's experiment provided answers to fundamental questions: Does electric charge change continuously? Can it be divided infinitely into smaller portions? Or is there a basic, elementary portion of charge that cannot be divided further?

In 1909, Robert A. Millikan conducted an experiment in which he studied the movement of oil droplets (aerosol) sprayed in the air between the plates of a flat capacitor. This experiment showed that the charges associated with the droplets were always multiples of a certain fundamental value, and consequently, the charge of a single electron is constant. Initially, its value was determined to be 1.33×10^{-19}C (in 1910) and later 1.592×10^{-19}C (in 1914). Currently, it is accepted as $1.602176634\times10^{-19}$C (since 2019).

In the original experiment, Millikan sprayed oil in the form of aerosol droplets, some of which became electrified in the air and entered the measurement chamber between two parallel capacitor plates in a vertical electric field. The movement of the oil droplets was observed through a microscope. The aerosol droplets fell due to gravity, overcoming the Stokes' drag force of the air, and the voltage in the capacitor additionally controlled the movement of the droplets depending on the charge on the plates and the sign of the charge carried by the droplets. By measuring their speeds at a known voltage on the capacitor plates, the charge on a selected droplet could be determined.

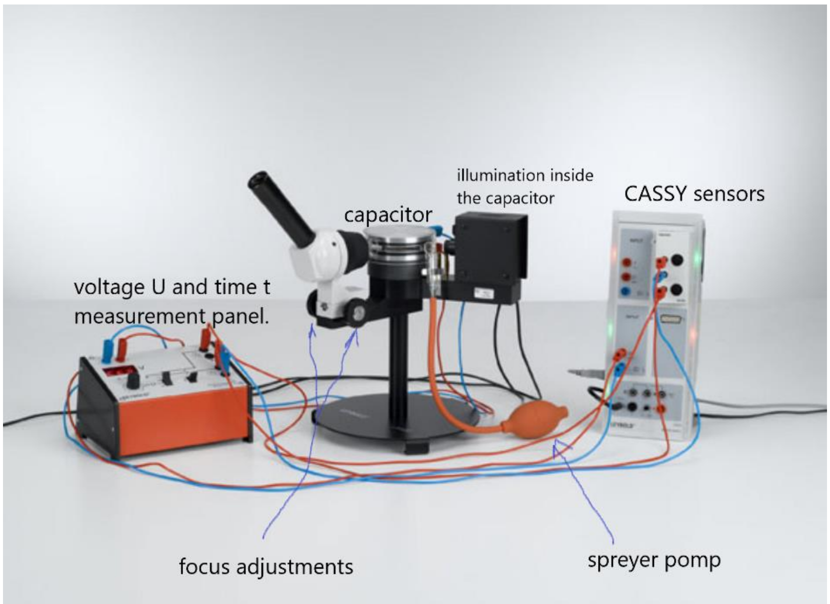

Fig.1. Set for conducting the experiment in the student laboratory.

Figure 1 shows the set used for the exercise. Round aluminum plates, which are the plates of a flat capacitor, are mounted on a stand. The voltage *U* between the capacitor plates can be adjusted with the potentiometer knob on the measurement panel. The capacitor is insulated with a plexiglass cover with holes – one near the tube with a capillary for spraying the aerosol and the other opposite the optical system to allow direct observation of the inside of the capacitor. The tube with the capillary, suspended on the side of the capacitor, is filled with oil and connected to a pump – its task is to spray oil from the capillary tip (very fine droplets) and allow the droplets to travel inside the capacitor. Those droplets that become electrified on the way between the capillary and the inside of the capacitor will be visible in the microscope eyepiece as moving differently than uncharged particles. In the setup at the station, charged droplets will move down faster than those falling only under the influence of the gravitational field.

Theory

When an oil droplet with radius r_0 falls with the velocity $-v_1$, this droplet is subject to Stokes friction, which leads to the upward directed force

$$F_1 = 6\pi\eta r_0 v_1 \quad (1)$$

η = viscosity of air.
When the oil droplet rises with the velocity v_2 in an external electrical field E, the downward directed force due to Stokes friction is

$$F_2 = -6\pi\eta r_0 v_2 \quad (2)$$

The difference between these two forces is exactly the force q_0E exerted by the applied electrical field E, i.e.

$$q_0 E = q_0 \frac{U}{d} = F_1 - F_2 = 6\pi\eta r_0 (v_1 + v_2) \quad (3)$$

And charge q_0 can be described as:

$$q_0 = \frac{6\pi \cdot \eta \cdot r_0 \cdot d \cdot (v_1 + v_2)}{U} \quad (4)$$

In order to determine the charge q_0, only the radius r_0 of the oil droplet under consideration is required, which, however, is easily obtained from the equilibrium of forces between its resultant gravitational force

$$F = -V\Delta\rho g \quad (5)$$

and the Stokes frictional force F_1 in the case of the falling droplet, where $\Delta\rho$ is the difference between the densities of oil and air.

Thus, we have:

$$0 = F + F_1 = -\frac{4}{3}\pi r_0^3 \Delta\rho g + 6\pi\eta r_0 v_1 \quad (6)$$

or

$$r_0 = \sqrt{\frac{9\eta v_1}{2\Delta\rho g}} \quad (7)$$

For a more precise determination of the charge q, it has to be taken into account that Stokes friction has to be corrected for very small radii r because these are of the order of magnitude of the mean free path of the air molecules.

The corrected formula for the frictional force, which depends on the air pressure p, reads

$$F = \frac{6\pi\eta r v}{1 + \frac{b}{rp}} \quad (8)$$

with b = 80 μm·hPa (constant).

With the abbreviation $A = b/p$, the corrected radius r is

$$r = \sqrt{r_0^2 + \frac{A^2}{4}} - \frac{A}{2} \quad (9)$$

and the corrected charge q is

$$q = \frac{q_0}{\left(1+\frac{A}{r}\right)^{1.5}} \quad (10)$$

Floating method

In this version of the experiment, the voltage U at the plate capacitor is adjusted such that a particular oil droplet floats, i.e. the rising velocity is υ_2=0. The falling velocity υ_1 is measured after switching off the voltage U at the capacitor.

Because of υ_2=0, the above formulas are slightly simplified.

However, υ_2=0 cannot be adjusted very precisely for fundamental reasons. Therefore the floating method leads to larger measurement errors and broader scattering in the frequency distribution than in the case of the following method.

Falling/rising method

In the second version, the two velocities υ_1 and υ_2 the voltage U are measured. This method makes possible more precise measured values than the floating method because the velocity υ_2 is really measured.

Exercise Execution

Check the elements of the set, turn on the measurement panel and CASSY sensors (plug in the power supplies or connect the power supply cables to the back or side of the housing), turn on the computer.

- Launch the Aparat application – it operates the camera mounted on a tripod.
- Launch the CASSYlab2 program from the desktop – it operates the CASSY sensors, which measure time t and voltage U. The program also has implemented formulas for calculating charge q, assuming that the distance traveled by the droplet is always s=20 divisions, which corresponds to 1 mm, and the measurement is always performed by measuring the time in which the droplet travels 20 divisions.
- Due to the limited duration of laboratory exercises and the difficulty in "seeing" the measurement field and droplets, measurements will be performed using the "floating" method for negatively charged droplets, falling in the gravitational field considering the retarding force of Stokes' drag.

Measurement Procedure (Floating Method)

Measurement Settings:

1. Set the eyepiece micrometer vertically – check if the vertical scale with divisions is clearly visible (if not, rotate the eyepiece lens holder (the end closer to the eye) until you clearly see the micrometer scale). The image sharpness is adjusted by turning the focus adjustment knobs on the sides of the eyepiece.
2. On the computer screen, split the view between the CASSYlab2 program window, which will show the obtained results, and the Aparat application if the camera is used.
3. Locate the potentiometer to change the voltage value on the capacitor, and the switches to measure voltage *U* and time *t* on the measurement panel. The current voltage will be displayed on the panel, but its more accurate value will be displayed in the CASSYlab2 program window.
4. If you are using the camera, set it so that you can see the falling aerosol droplets on the monitor (the camera does not have focus and zoom settings). The light should be turned off when using the camera to avoid additional reflections that could disturb the image. The eyepiece axis and the camera axis should be parallel, and the camera should be as close to the eyepiece as possible. To see the aerosol droplets, occasionally squeeze the sprayer pump to spray a new batch of droplets.

Then:

a) First, set the switches *U* and *t* to the lower position.

b) Turn on the voltage on the capacitor using the *U* switch and adjust it with the rotary potentiometer (400-600 V), observing the droplets on the screen or in the eyepiece so that the selected oil droplet falls at a speed of about 1-2 scale divisions per second. Then reduce the voltage, checking if the droplet stops falling.

c) Turn off the voltage on the capacitor using the *U* switch.

d) The droplet will start to fall. When it reaches the height of the selected scale division, start the time measurement using the *t* switch.

e) When the oil droplet falls by 20 scale divisions (which corresponds to 1 mm), stop the time measurement using the *t* switch and turn the voltage on the capacitor back on using the *U* switch.

f) Enter the measured values of falling time t_i and voltage U_i into the computer using the stopwatch button (Single set measurement) in the taskbar at the top, the charge *q* will be calculated automatically and entered into the graph. During subsequent measurements, the graph will be automatically updated for the consecutively measured charge values. The graph, where the measurement data will be entered, is selected from the drop-down menu of the CASSYlab2 program → New data measurements. This graph is a histogram of scaled charge values, i.e., on the horizontal axis it has the value $q_{\text{measured}}/e_{\text{tabulated}}$, and on the vertical axis the number of occurrences of a given value from the histogram interval. A successful experiment should show clear maxima of occurrences for values 1, 2, 3, etc.

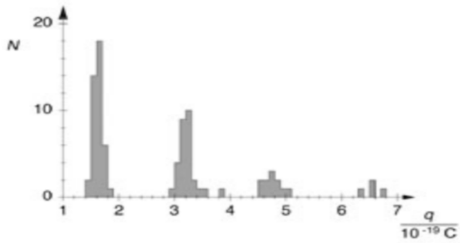

Fig.2. Example histogram with measurement data (on the horizontal axis calculated charge values, not scaled to the elementary charge)

g) Repeat the measurement for other oil droplets. To draw any conclusions from the experiment, 20-30 measurements should be performed for different droplets.

Notes:

- If oil droplets with a small charge are selected, statistical significance will be achieved faster. Oil droplets carrying a small charge are identified by their small size and slower movement in the electric field, compared to significantly faster movement of droplets carrying a larger charge. They will be visible as sharply outlined, shiny dots clearly moving against the background of slowly falling, uncharged aerosol droplets.
- Very slowly falling and non-reacting droplets are neutral (uncharged) droplets, it makes no sense to perform measurements as the result will be close to zero.
- It is not recommended to measure fast-moving but heavily blurred droplets – these are droplets whose path seen in the eyepiece is heavily distorted by optical elements and as such unreliable.
- If the local air pressure differs significantly from 1013 hPa, the air pressure should be appropriately changed in the formula defining the correction parameter AA. However, the displayed example values (calculated for the above pressure) will then be inaccurate.
- If there is not enough time during the class to observe about 20-30 oil droplets, an example (from the CASSYlab2 program resources) with measured values (about 20) can be introduced instead of the settings, and new measured values will then appear on the histogram as supplementary red markers, thus confirming and supplementing previous measurements.
- Remember to save the measurement data in a format readable outside the CASSYlab2 program.

Analysis During the analysis, based on the collected data, determine the average values in the measured frequency distribution of specific charge values and check if the relationship q=n·e$q=n\cdot e$ (with e=1.6022×10−19$e=1.6022\times10-19$ C) is confirmed. Discuss the difficulties encountered during the measurements. Consider what modern devices or meters could significantly improve the obtained measurement results.