

Exercise 0: Determination of the acceleration due to gravity, g , using a simple pendulum

In this experiment, a simple pendulum consists of a small ball hanging from an inextensible string of negligible mass, the length of which is much greater than the size of the ball.

The acceleration due to gravity will be determined using the formula for the period of oscillation of a simple pendulum:

$$T = 2\pi \sqrt{\frac{L}{g}} \quad g = \frac{4\pi^2 L}{T^2}$$

where T is the period of oscillation (the time required for the pendulum to move from its maximum displacement on one side, through its maximum displacement on the opposite side, and back to the left again), L is the length of the pendulum (the distance from the point of suspension to the centre of mass of the bob), and g is the gravitational acceleration, whose expected value in Kraków is **9.811 m/s²**.

The gravitational acceleration will be determined using two different methods

Method I

$$T = 2\pi \sqrt{\frac{L}{g}} \quad \text{czyli} \quad g = \frac{4\pi^2 L}{T^2} \quad (\text{working equation})$$

Let us measure the length of the pendulum $L = \dots\dots\dots$ (remember to include the unit!)

The smallest scale division of the instrument is $\Delta L = \dots\dots\dots$ (again, remember the unit!)

What should be taken as the maximum uncertainty $u(L)$ (i.e. the uncertainty associated with length L)?

It may be equal to the smallest scale division of the instrument, or larger if we consider the determined length to be less certain. Therefore, I assume $u(L) = \dots\dots\dots$ (unit!)

(this is a Type B uncertainty associated with a single measurement)

(If, apart from the instrument scale division, no additional information is available, we may assume the standard uncertainty (Type B) of the length measurement to be $\dots\dots\dots$ (unit!))

One may also determine **the relative uncertainty** of the pendulum length measurement: $\frac{u_B(L)}{L} = \dots\dots\dots$ (no unit!)

Next, measure the time required for 10 periods (10 complete oscillations). Then convert it to the duration of a single period and calculate the mean period as well as its *standard uncertainty (Type A)* (i.e. the uncertainty determined from a series of measurements).

set number	duration of 10 periods	Duration of the single period	Squared deviation from the average value	The mean value of the period: $T_{\text{sr}} =$ The standard uncertainty $u_A(T_{\text{avg}})$ (of the initial preliminary measurement) is calculated using: $u_A(T_{\text{sr}}) = \sqrt{\frac{\sum (T_i - T_{\text{sr}})^2}{N(N-1)}} =$ (we divide the sum of the squared deviations by the number of measurements T_{sr} multiplied by N-1) (then we take the root of the entire expression) The standard uncertainty of the mean period is therefore equal to: $\dots\dots\dots$
i	$10 T_i [\text{s}]$	$T_i [\text{s}]$	$(T_i - T_{\text{sr}})^2$	
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
$N = 10$		$T_{\text{sr}} =$	$\sum_{i=1}^{10} (T_i - T_{\text{sr}})^2 =$	

Relative uncertainty of the mean period: $\frac{u_A(T_{\text{sr}})}{T_{\text{sr}}} =$

Relative uncertainty of the mean period: $g = \frac{4\pi^2 L}{T_{\text{sr}}^2} =$

To determine the uncertainty of the calculated gravitational acceleration, we will use either the relative uncertainty **propagation law** or the general uncertainty propagation law.

From the law of propagation of relative uncertainty: $\frac{u(g)}{g} = \sqrt{\left(\frac{u_B(L)}{L}\right)^2 + \left(2 \cdot \frac{u_A(T_{\text{sr}})}{T_{\text{sr}}}\right)^2} = \dots\dots\dots$ that is

$$u(g) = g \cdot \sqrt{\left(\frac{u_B(L)}{L}\right)^2 + \left(2 \cdot \frac{u_A(T_{\text{sr}})}{T_{\text{sr}}}\right)^2} = \dots\dots\dots \approx \dots\dots\dots$$

Or from the law of uncertainty propagation, with the general formula: $u(g) = \sqrt{\sum_k \left(\frac{\partial g}{\partial x_k} u(x_k) \right)^2}$, which in this case (k=2) yields $u(g) = \sqrt{\left(\frac{\partial g}{\partial L} u(L) \right)^2 + \left(\frac{\partial g}{\partial T} u(T_{sr}) \right)^2} = \sqrt{\left(\frac{4\pi^2}{T_{sr}^2} u(L) \right)^2 + \left(\frac{-8\pi^2 L}{T_{sr}^3} u_A(T_{sr}) \right)^2} = \dots \approx \dots$

We round the uncertainty and the value appropriately (to two significant digits):

$$g = \quad , \quad u(g) =$$

Method II

This time, we will determine the gravitational acceleration by measuring oscillation periods for various pendulum lengths.

$T = 2\pi \sqrt{\frac{L}{g}}$ We can rearrange this formula and express it as a linear relationship: $T^2 = \frac{4\pi^2}{g} L$ (working formula)

where T is the oscillation period, L is the pendulum length, and g is the acceleration due to gravity.

We measure the pendulum length and record it in the table, then measure the duration of 10 oscillation periods and record that as well. Due to time constraints, for each of the three different pendulum lengths, we measure the duration of 10 periods only three times. We will use the data from the first measurement method as the fourth data set.

Set number	Length of the pendulum	Duration of 10 periods	Duration of the single period	Average period for a given L	the square of the oscillation period	Type of linear relationship:
i	$L_i [\text{cm}]$	$10 T_i [\text{s}]$	$T_i [\text{s}]$	$T_{\text{sr}} [\text{s}]$	$T_i^2 [\text{s}^2]$	$y = Ax + B$
1				$T_1 =$	$T_1^2 =$	In this case:
1						$y \rightarrow T^2 \quad x \rightarrow L \quad A \rightarrow \frac{4\pi^2}{g}$
1						$T^2 = \frac{4\pi^2}{g} L$
2				$T_2 =$	$T_2^2 =$	We have 4 datasets
2						$y \quad i \quad x:$
2						$T_1^2 \quad i \quad L_1$
3				$T_3 =$	$T_3^2 =$	$T_1^2 \quad i \quad L_1$
3						$T_1^2 \quad i \quad L_1$
3						$T_1^2 \quad i \quad L_1$
4 (from previous method)				$T_4 =$	$T_4^2 =$	

Prepare a linearised graph of T^2 as a function of L .

Using these four points with coordinates (L_i, T_i^2) we will prepare a linearised graph of T^2 as a function of L . (It is worth remembering to enter the data into Excel in appropriately converted base units—here, meters and squared seconds—and using Excel's LINEST(or REGLINP) statistic function, we will fit a regression line to the points: $T_i^2 = A L_i + B$, that is, Excel will return the values of the line parameters A and B , as well as their uncertainties $u(A)$ and $u(B)$. For our purposes, A and $u(A)$ are the important values.

The obtained coefficient is $A = \dots$ (unit—determined by noting that the slope of the line is the tangent of the angle of inclination, i.e., in this case, $\Delta T^2 / \Delta L$)

$u(A) = \dots$ (unit!—same as above)

Since $A = \frac{4\pi^2}{g}$, we calculate that $g = \frac{4\pi^2}{A} = \dots = \dots$ (unit!)

We calculate the uncertainty using the law of propagation of uncertainty (which results in a fairly simple formula in this case, as the only quantity with associated uncertainty is the fitted coefficient A):

$$u(g) = \sqrt{\left(\frac{4\pi^2}{-A^2} \right)^2 (u(A))^2} = \frac{4\pi^2 u(A)}{A^2} = \dots (\text{unit}) \approx \dots (\text{unit})$$

Rounded the uncertainty and the value appropriately:

$$g = \quad , \quad u(g) =$$

Conclusions: