EXPERIMENT NO.1 — PHYSICAL PENDULUM [1)]

Purpose:

Observations of the oscillatory motion of a physical pendulum. Determination of the moment of inertia of rigid bodies (simple geometrical shapes).

Introduction:

A mathematical pendulum consists of a point-like mass m attached to a weightless cord of a length l. **A physical pendulum** (cf. Fig.1-1) is a rigid body, which can rotate around an axis 0. The axis 0 is at a distance l from the center of mass of the body C. In the limit of a negligible size of the body a physical pendulum becomes a mathematical one.
If deflected laterally from the equilibrium position ($\theta = 0$) through an angle θ, the pendulum performs the oscillatory motion. The equation governing the motion is the 2nd Law of Dynamics, i.e.:

$$I_0 \frac{d^2\theta}{dt^2} = -mg\,l\,\sin\theta, \tag{1.1}$$

where I_0 is the pendulum rotational inertia, $d^2\theta/dt^2 = \epsilon$ is the pendulum's angular acceleration, and $mgl\sin\theta$ is the absolute value of the restoring torque.
For a small initial deflection, $\sin\theta \approx \theta$ and equation 1.1 becomes

$$\frac{d^2\theta}{dt^2} + \omega_0^2\theta = 0, \tag{1.2}$$

with $\omega_0^2 \stackrel{\text{def}}{=} \frac{mgl}{I_0}$. The solution of Eq. 1.2 is the usual

$$\theta(t) = \theta_{max}\cos(\omega_0 t + \alpha), \tag{1.3}$$

with constants θ_{max} and α that can be determined from the initial conditions. The solution represents the simple harmonic motion with the period, T, equal to

$$T = 2\pi\sqrt{\frac{I_0}{mgl}}. \tag{1.4}$$

The experiment consists in measuring the period T, the axis-center-of-mass distance l and the mass m. Once these quantities are known equation 1.4 can be solved for I_0. Note that this quantity is the rotational inertia about an axis of rotation that passes through the point of suspension, 0. The inertia about an axis of rotation that passes through the center-of-mass , I_C, can be calculated from the **parallel-axis theorem**:

$$I_0 = I_C + ml^2. \tag{1.5}$$

It is worth noting, that in the limit of a point-like mass $I_C = 0$, so $I_0 = ml^2$, and inserting this value into equation 1.4 we recover the well known formula for the oscillation period of the mathematical pendulum.

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1) compiled by Andrzej Lenda on the basis of: "PRACOWNIA FIZYCZNA" WFiTJ, Andrzej Zięba (editor), skrypt AGH 1529

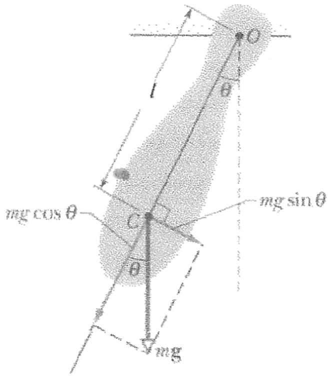

Figure 1-1: A physical pendulum.

For simple geometrical shapes (like those used in the experiment) the rotational inertia can be calculated analytically, using the basic definition of the inertia that describes this quantity as an integral of all the point-like mass-particles dm multiplied by the squares of their distances from the axis of rotation, r^2:

$$I = \int_m r^2 dm \tag{1.6}$$

with the integration carried out over the whole volume of the body. For a rod (mass m, length L) and for a ring (mass m, inner radius R_w, outer radius R_z) the I_C values are (cf. Fig.1-2 – the rotation axis is perpendicular to the figure plane):

$$I_C^{\mathrm{rod}} = \frac{m}{12}L^2, \qquad I_C^{\mathrm{ring}} = \frac{m}{2}\left(R_w^2 + R_z^2\right). \tag{1.7}$$

Thus, the values of I_C measured experimentally (via the measurement of T) can be confronted with the values calculated accordingly to equation 1.7. This requires experimental determination of mass and geometrical parameters of the pendula, usually with the help of simple tools.

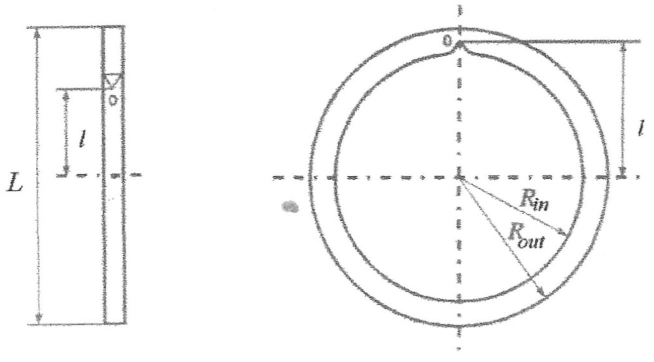

Figure 1-2: Rod and ring and their parameters to-be-measured.

Experimental Setup

The experimental setup consist of a rod and ring, which can be suspended on an appropriate stand. The measuring devices are: simple weighing scale (pan scales), slide caliper, measuring tape and timer.

A. Determination of inertia for ring and rod:

Experimental Procedure

1. Weigh the rod and the ring, using the pan scales.
2. Measure the lengths: a, l and radii R_w, R_z using the measuring tape and slide caliper.
3. After hanging the rod (ring) on the stand, make it oscillate (the initial deflection should be no greater than few degrees) and measure the time of 30-40 oscillations. Repeat this measurement 10 times and calculate the period T as an arithmetic mean.

Report

1. Calculate (using equations 1.4 and 1.5) the rotational inertia: I_0 and I_C for the ring and for the rod.
2. Calculate the errors of determination.
3. Calculate the corresponding I_C values and their errors using the formulae 1.7.
4. Compare the I_C values obtained using these two methods.