

PHYSICS LABORATORY

Experiment No. 11 - Young's Modulus Aleksandra Orzechowska

Purpose: Determination of the Young's modulus using the static method.

Introduction:

Objects can deform while external forces act on them. Deformation of solids can be explained in terms of concepts of stress and strain. Stress is quantity that is proportional to the force causing a deformation. It is the external force acting on object per unit cross-sectional area. Strain is a measure of the degree of deformation. It is found that for sufficiently small stresses, strain is proportional to stress. The constant of proportionality depends on material being deformed and on the nature of deformation and it is called elastic modulus. The elastic modulus is therefore the ratio of the stress to the resulting strain.

$$\text{Elastic modulus} = \frac{\text{stress}}{\text{strain}}$$

One can consider three types of deformations as well as define the elastic modulus for each :

1. Young's modulus- (elasticity in length) which is the measure of the resistance of solid to a change in its length.
2. Shear modulus- (elasticity of shape, modulus of rigidity), which measure the resistance to the motion of the planes of a solid sliding past each other.
3. Bulk modulus- (volume elasticity), which measures the resistance of solids or liquids to changes in their volume.

Here, we will focus only on the first case. Consider a long bar of cross-sectional area A and initial length L_0 . (Figure 1). When an external force is applied perpendicular to the cross section, internal forces in the bar resist distortion. However, the bar attains an equilibrium in which its length is greater than L_0 .

Young's modulus is defined as a combination of the two terms, tensile stress and tensile strain respectively.

$$E = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_0}$$

Tensile stress (σ) is the ratio of the magnitude of the external force F to the cross-sectional area A . The tensile strain (ϵ) is the ratio of the change in length ΔL to the original length L_0 . Young's modulus is typically used to characterize a wire (or rod) stressed under either tension or compression. Typical values are given in Table 1. [1].

Table 1. Typical values for elastic modulus

Substance	Young's modulus (N/m ²)	Shear Modulus (N/m ²)	Bulk modulus (N/m ²)
Tungsten	$35 \cdot 10^{10}$	$14 \cdot 10^{10}$	$20 \cdot 10^{10}$
Steel	$20 \cdot 10^{10}$	$8,4 \cdot 10^{10}$	$6 \cdot 10^{10}$
Copper	$11 \cdot 10^{10}$	$4,2 \cdot 10^{10}$	$14 \cdot 10^{10}$
Brass	$9,1 \cdot 10^{10}$	$3,5 \cdot 10^{10}$	$6,1 \cdot 10^{10}$
Aluminium	$7,0 \cdot 10^{10}$	$2,5 \cdot 10^{10}$	$7,0 \cdot 10^{10}$
Glass	$6,5-7,8 \cdot 10^{10}$	$2,6-3,2 \cdot 10^{10}$	$5,0-5,5 \cdot 10^{10}$
Quartz	$5,6 \cdot 10^{10}$	$2,6 \cdot 10^{10}$	$2,7 \cdot 10^{10}$

The elastic limit of a substance is defined as a maximum stress that can be applied to the substance before it becomes permanently deformed (Figure 1). Initially, a stress-strain curve is a straight line (elastic behaviour) . As the stress increases, the curve is no longer straight. When the stress exceeds the elastic limit, the substance is permanently distorted and does not return to its original shape after the stress is removed. In this situation, the shape of the object is permanently changed. As the stress is increased even further, the material ultimately breaks.

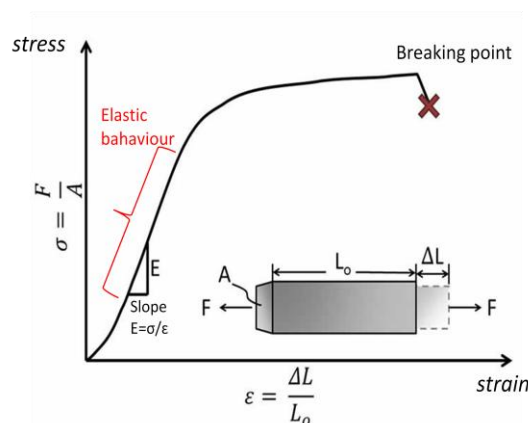


Figure 1: A typical stress-strain diagram for a homogenous material that is tested to failure (X). The slope of the linear portion of the curve represents the Young's Modulus ($E = \sigma/\epsilon$). According to [2].

Experimental setup:

1. System for measuring the extension of the wire as the result of the constant strength action equipped with the micrometric gauge.
2. Set of different weights.
3. Micrometric screw.
4. Gauge meter.

Experimental procedure:

1. Measure the length of the steel wire. You can use the gauge meter which is fixed to the right arm of the stand.
2. Release blockade of the lever (horizontal position) and then fix the wire in the stand by the caps which are attached to the both ends of the wire.
3. Measure the diameter of the wire while it is loaded (1 or 2 kg). Make the measurements at three different points along its length using micrometric screw.
4. Remove the weights from the scale and reset the micrometric gauge. Then place the subsequent weights on the scale and note (in Table 2 and 3) the strength and extension of the wire. Remember that maximum weight for steel and brass wire is 10 and 6 kg respectively. Repeat this experiment by removing the following weights (diminishing the strength) from the scale and note the values shown by the micrometric gauge.
5. Repeat the whole procedure (step 1-4) for the brass wire.

Report:

1. Make a graph of the mean wire extension ΔL as a function of the tensile strength F .
2. Prepare a linearised graph of ΔL as a function of F and fit the obtained points with a line $\Delta L = aF + b$. Calculate a and its uncertainty $u(a)$.

3. From the calculated slope $a = \frac{L_0}{EA}$, find the Young's modulus of the steel and brass wire.

Remember, that area is given as: $A = \frac{\pi d^2}{4}$, hence Young's modulus can be determined from the equation $E = \frac{4L_0}{\pi d^2 a}$

4. Calculate the $u(E)$ using the propagation of relative uncertainty:

$$\frac{u(E)}{E} = \sqrt{\left(\frac{u(L_0)}{l}\right)^2 + \left(-2\frac{u(d)}{d}\right)^2 + \left(-\frac{u(a)}{a}\right)^2}$$

5. Evaluate agreement with the tabular value.

Questions for discussion:

1. Hook's law
2. Young's modulus- definition and unit
3. Stress-versus-strain curve for an elastic solid.

References:

[1] R.A.Serway, R.J.Beichner, *Physics for scientists and engineers with modern physics*, Saunders College Publishing, Orlando, USA, 2000.

[2] <http://www.med.upenn.edu/pcmd/BCTestingMethods.shtml>

Results:

1. Steel wire:

length L_0 :

wire diameter d (3 measurements): $u(d)$ = ...

Table 2. Experimental data obtained for steel wire.

m [kg]	F [N]	$L_{(\uparrow)}$ [mm]	$L_{(\downarrow)}$ [mm]	ΔL

2. Brass wire:

length L_0 :

wire diameter d (3 measurements): $u(d)$ = ...

Table 3. Experimental data obtained for brass wire.

m [kg]	F [N]	$L_{(\uparrow)}$ [mm]	$L_{(\downarrow)}$ [mm]	ΔL