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 EXPERIMENT No.13 — VISCOSITY COEFFICIENT
 

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**Purpose:**

Determination of the viscosity coefficient of a real liquid using the Stokes' method (falling sphere).

**Introduction:**

When studying the real fluid (liquid or gas) flow one observes some frictional phenomena (energy dissipation in a volume of liquid). This phenomena occur in the whole volume of fluid and bear the name of **intrinsic friction** or **viscosity**.

In order to visualize this phenomenon, let us consider two plates immersed in a viscous liquid (or gas). If one plate is moving with respect to the other one, the frictional braking force (to be overcome by an external applied force, if the motion is to be continued)  $F$  is proportional to the area  $S$  of each of the plates, and to the relative velocity  $v$  and inversely proportional to the inter-plate distance  $d$ . The coefficient of the proportionality is **the viscosity coefficient**  $\eta$ , a material constant that depends on the nature of liquid (or gas) and temperature, whose units are Pa·s. The equation relating all the introduced quantities takes the form (cf. Fig.13-1):

$$F = \eta \frac{S v}{d}. \quad (13.11)$$

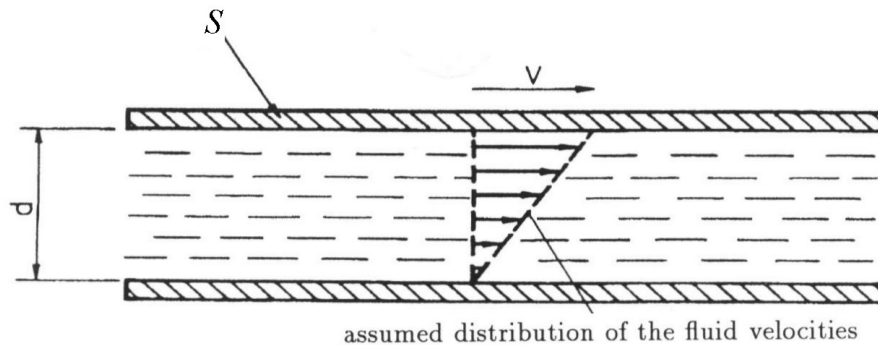


Figure 13-1: Motion of two plates in a viscous fluid.

assumed distribution of the fluid velocities

**Fall of a sphere in a viscous liquid**

Consider a sphere falling under the action of gravity in a viscous liquid. If the velocity of the fall is relatively small, the motion can be treated as a **laminar** one, i.e. no turbulences or vortices accompany the sphere's motion. Besides the gravity, the forces acting on the sphere are: the hydrostatic lift  $F_w$  and the viscosity force (viscosity drag)  $F_o$ . The latter can be shown to be equal to

$$F_o = 6\pi\eta r v, \quad (13.12)$$

where  $v$  is the velocity of the falling sphere and  $r$  is its radius. The above formula (the so-called Stokes' formula) applies if the sphere moves through an unlimited volume of liquid. For a sphere falling in a cylinder (along its axis) of the radius  $R$  the above formula has to be slightly modified and it becomes

$$F_o = 6\pi\eta rv \left(1 + 2.4\frac{r}{R}\right) \equiv K v, \quad (13.13)$$

with  $K = 6\pi\eta r(1 + 2.4r/R)$ .

The hydrostatic force being equal to the product of the sphere volume  $V$ , liquid's density  $\rho$  and gravity  $g$ , the equation of motion (Cf. Fig.13-2) for the sphere's velocity becomes:

$$m \frac{dv}{dt} = mg - \rho g V - K v. \quad (13.14)$$

Note, that the two forces ( $mg$  and  $\rho g V$ ) are constant terms. This equation can be readily

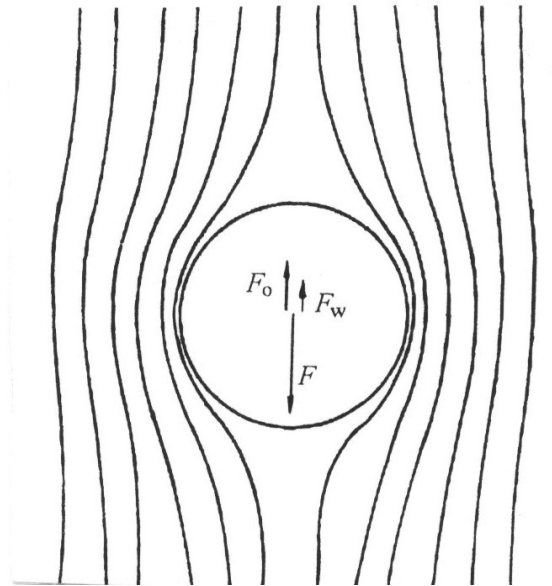


Figure 13-2: Sphere falling in a viscous fluid.

solved for the velocity  $v$ :

$$v(t) = v_{eq} + (v_0 - v_{eq})e^{-tK/m} \equiv v_{eq} + (v_0 - v_{eq})e^{-t/\tau}. \quad (13.15)$$

The time constant  $\tau$  is equal to the ratio  $m/K$  and may be perceived as a "natural" time-unit;  $v_0$  is the initial sphere's velocity, at  $t = 0$ , and  $v_{eq}$  is the velocity attained by a sphere for which the gravity force and hydrostatic lift difference becomes exactly balanced by the viscosity drag:

$$K v_{eq} = mg - \rho g V. \quad (13.16)$$

As the exponential term in 13.15 becomes negligible after  $t \approx 3\tau$  the observed sphere's motion can be approximated as a uniform one with the velocity

$$v(t) = v_{eq} = \frac{(m - \rho V)g}{6\pi\eta r \left(1 + 2.4\frac{r}{R}\right)}. \quad (13.17)$$

A simple transformation of equation 13.7 allows to calculate  $\eta$  if the remaining quantities are known (measured). The  $v_{eq}$  velocity can be measured as the quotient of the distance  $l$  (cf. Fig.13-4) traveled by the sphere (once the uniform motion has set in) during the time  $T$ :

$$v_{eq} = \frac{l}{T}. \quad (13.18)$$

A graph of equation 13.7 is shown in Fig.13-3.

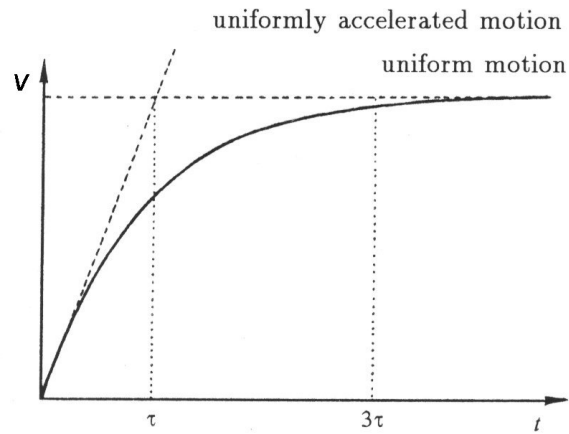
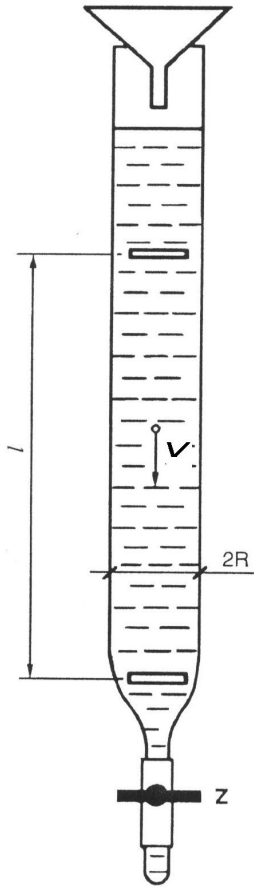


Figure 13-3: Sphere's velocity versus time.

### Experimental Setup

The experimental setup (cf. Fig.13-4) consist of a glass cylinder filled by a viscous liquid (glycerin,  $\text{CH}_2\text{OH}\cdot\text{CHOH}\cdot\text{CH}_2\text{OH}$ ). Two horizontal bars determine the  $l$  distance that has to be used for measuring  $v_{eq}$ .



### Experimental

1. Weigh the balls using either an analytical or torsion balance.
2. Measure the ball diameters  $2r$  using a micrometer screw.
3. Measure the  $l$  distance and the cylinder inner diameter  $2R$ .
4. Read out the temperature of the liquid.
5. Measure the time  $T$  of the fall for all spheres.

### Report

1. Calculate (using equation 13.17) the viscosity coefficient,  $\eta$ .
2. Calculate the error of the  $\eta$  determination.
3. Calculate the values of the Reynolds number for all the spheres used.

Reynolds number  $\text{Re}$  is given by the equation

$$\text{Re} = \frac{vL\rho}{\eta}, \quad (13.19)$$

Figure 13-4: Experimental setup for measuring  $v_{eq}$ .

where the notation is as above.  $L$  is a linear dimension of the body perpendicular to direction of the motion. For a sphere  $L = 2r$ .

### Questions for discussion:

- Viscosity; viscosity coefficients (units!)
- Archimedes Law (hydrostatic lift). Why some bodies float and others sink?
- Forces acting on a ball moving under the gravity through a viscous liquid. Equation of motion. Its solution (for the velocity).