

Purpose:

Wheatstone bridge as an example of Kirchhoff's Rules applied to the description of complex electric circuits. Measuring unknown resistances, and their parallel or serial combinations.

Introduction: Finding the currents and voltages in all parts of electric circuits is crucial for the process of design of such systems. Solving that kind of problems involves the following laws:

- ratio of the voltage between ends of a conductor to the current is constant (Ohm's law),
- the algebraic sum of the currents into any junction is zero (Kirchhoff's junction rule),
- the algebraic sum of the potential differences in any loop must equal zero (Kirchhoff's loop rule).

These laws lead to a system of independent linear algebraic equations allowing to find the needed currents. (Please note that the Ohm's law is not valid in nonlinear devices, e.g. diodes).

A Wheatstone bridge is a system used to measure (or compare) resistances. It is build of four resistances: R_x , R_2 , R_3 , R_4 and a galvanometer having resistance R_5 . The bridged is connected to a voltaic cell or other power supply (Fig. 32-1) with emf ε and internal resistance r . Let I be the current flowing from the cell, and the currents in the sections AB , AD , BC , DC and BGD be I_1 , I_2 , I_3 , I_4 and I_5 , respectively. The are four junctions in the circuit: A, B, C, D , and we can use Kirchhoff's junction rule for three of them, following the direction of currents denoted by the arrows in Fig. 32-1:

$$\begin{aligned} A: \quad I - I_1 - I_3 &= 0, \\ B: \quad I_1 - I_2 - I_5 &= 0, \\ D: \quad I_5 + I_3 - I_4 &= 0. \end{aligned} \tag{32.1}$$

Another three equations may be written using Kirchhoff's loop rule for $ABDA$, $BCDB$ and $ACEA$:

$$\begin{aligned} ABDA: \quad I_1 R_x + I_5 R_5 - I_3 R_3 &= 0, \\ BCDB: \quad I_2 R_2 + I_4 R_4 - I_5 R_5 &= 0, \\ ACEA: \quad I_3 R_3 + I_4 R_4 + I r &= \varepsilon. \end{aligned} \tag{32.2}$$

Knowledge of the emf ε and all the resistances allows to calculate the currents.

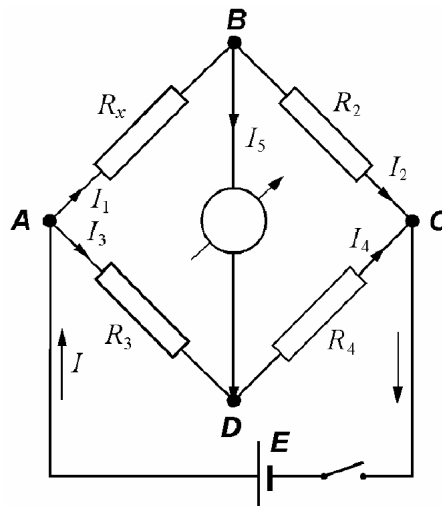


Figure 32-1: Wheatstone bridge

The Wheatstone bridge is used to compare resistances by means of balancing the bridge, i.e. choosing the resistances to make the potentials at the points B and D equal ($V_B = V_D$). This leads to zero current through the galvanometer ($I_5 = 0$), and from the second and the third equations of (32.1):

$$I_2 = I_1, \quad I_3 = I_4, \quad (32.3)$$

while from the first and the second equations of (32.2):

$$I_1 R_x = I_3 R_3, \quad I_2 R_2 = I_4 R_4, \quad (32.4)$$

which leads to an expression allowing to find R_x experimentally:

$$R_x = R_2 \frac{R_3}{R_4}. \quad (32.5)$$

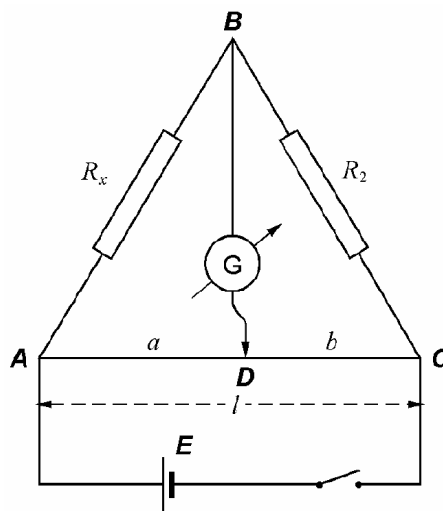


Figure 32-2: Wheatstone bridge – experimental setup with a resistance wire.

Version of the Wheatstone bridge used during this experiment is shown in Fig. 32-2. Current from the power supply splits in A . One part of it flows through the serial combination of R_x and R_2 , and the wire AC carries the rest of it. Changing the position of D modifies the ratio of resistances R_3 and R_4 .

There is no current along BGD if

$$\frac{R_x}{R_2} = \frac{R_{AD}}{R_{DC}}. \quad (32.6)$$

R_{AD} and R_{DC} are resistances of sections of the same homogeneous wire which have lengths a and b (Fig. 32-2), and thus

$$R_{AD} = \rho \frac{a}{S}, \quad R_{DC} = \rho \frac{b}{S}, \quad (32.7)$$

where S is the area of wire's cross section, and ρ is its resistivity. Substituting (32.7) into (32.6) gives

$$\frac{R_x}{R_2} = \frac{a}{b}. \quad (32.8)$$

The total length of the wire is $l = a + b$, and then $b = l - a$. Finally, the formula allowing to calculate unknown resistance R_x if we know the resistance R_2 and lengths l and a , is:

$$R_x = R_2 \frac{a}{l - a}. \quad (32.9)$$

Precision of the measurements with the Wheatstone bridge containing a resistance wire depends mainly on the accuracy of the measurement of a . According to the error propagation formula,

$$u(R_x) = \left| \frac{dR_x}{da} u(a) \right| = R_2 \frac{l}{(l-a)^2} u(a), \quad (32.10)$$

and the relative uncertainty of the measured resistance is

$$\frac{u(R_x)}{R_x} = \frac{R_2 \frac{l}{(l-a)^2} u(a)}{R_2 \frac{a}{l-a}} = \frac{l}{l-a} \frac{u(a)}{a}. \quad (32.11)$$

To find the value of a , for which the relative uncertainty has the smallest possible value, the derivative of (32.11) with respect to a has to be equated to zero. The result of such procedure is that

$$-\frac{l(l-2a)}{a(l-a)^2} \frac{u(a)}{a} = 0, \quad (32.12)$$

and the solution of this equation is $a = l/2$. It means, that for the most accurate results, we should choose R_2 allowing for balancing the bridge when D is possibly close to the middle of the wire.

Experimental Setup

The experimental setup is presented in Fig. 32-2 and contains:

- a wire AC placed along a millimetre scale, with a sliding contact at the point D allowing to change a and b ,
- a resistance decade box R_2 ,
- a set of resistances R_x mounted on a Plexiglas plate,
- a galvanometer G with adjustable sensitivity (to indicate zero current through BGD),
- a 3A/30V power supply.

Experimental Procedure

1. Assemble the circuit as shown in Fig. 32-2. DO NOT switch the power supply on before it is checked by a member of the laboratory staff.
2. Balance the bridge:
 - (a) place the sliding contact in the middle of the wire,
 - (b) set R_2 to roughly balance the bridge,
 - (c) balance the bridge by changing the position of the sliding contact.
3. Measure and note the values of R_2 and a . Repeat it several times, slightly changing R_2 .
4. Repeat the whole procedure for several chosen resistances R_x and their parallel and serial combinations.

Report

1. Calculate the values of R_x and serial/parallel combinations for all conducted measurements. Calculate the average value and uncertainty for each R_x and for each combination.
2. Check if the results obtained for parallel and serial combinations agree (within the uncertainty – use error propagation!) with the values calculated directly from the formulas describing the total resistance of resistors in parallel or resistors in series.

Questions for discussion:

1. Discuss Kirchhoff's laws.
2. Derive the formulas for the equivalent resistance of two resistors R_1 and R_2 connected in series and parallel. What is resistivity and electrical conductivity?
3. What does the resistance of a given section of a conducting wire depend on? Discuss the dependence of the electrical resistance of metals on temperature.
4. Define and discuss the concepts of electric current and charge. Provide the definitions of their corresponding units.
5. Define and discuss the concepts of voltage and electrical resistance. Provide the definitions of their corresponding units.